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Parametric nonlinear programming problems using numerical bifurcation and continuation methods have been investigated with respect to design optimization applications. New combinatorial optimization techniques to solve a central problem of multi-target tracking, the partitioning of observations into tracks and false alarms, have been developed.

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by

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Abstract

This report describes some of the problems, achievements, and directions of investigation of three areas of research. The first is the investigation of parametric nonlinear programming problems using numerical bifurcation and continuation methods and with applications to design optimization and parametric control systems. The second part centers on investigation of various numerical methods for the solution of nonlinear optimal control problems. The analysis of convergence in infinite dimensional spaces, discretizations, and numerical implementations are in progress for Newton's, penalty, augmented Lagrangian, and interior point methods. The third part ot this research program is the development of combinatorial optimization techniques to solve the central problem of multitarget tracking, i.e., the data association problem of partitioning observations into tracks and false alarms. The problem formulation, algorithm design, and real-time solution involve techniques from probability and information theory, system identification, filtering, control systems, combinatorial optimization, and advanced computer architectures, including massively parallel computers. The data association problem for general multi-target tracking problems is posed as a class of multi-dimensional assignment problems. The algorithms under development are based on a recursive Lagrangean relaxation scheme, construct high quality suboptimal solutions in real-time, and use a variety of techniques ranging from two dimensional assignment algorithms, a conjugate subgradient method for the nonsmooth optimization, graph theoretic properties for problem decomposition, and a branch and bound technique for small solution components. These algorithms are being implemented on massively parallel computer architectures for increased performance and real-time identification.

TABLE OF CONTENTS

1 Introduction

This report describes some of the problems, achievements, and directions of investigation of three areas of research. The first is the investigation of parametric nonlinear programming problems using numerical bifurcation and continuation methods with applications to design optimization and parametric control systems, and represents a potential for a real extension of our understanding of basic phenomena, global sensitivity, robustness, and multiplicity of solutions in much the same way that these theoretical and numerical techniques have helped the understanding of dynamical systems and nonlinear equations. Thus the objective in this aspect of the research program is to develop the analytical and numerical techniques to map out regions of qualitatively different behavior and to locate the "stability" boundaries of these regions in parameter space. The latter is important because drastic changes in the optimum occur in the presence of singularities which, in turn, define these "stability" boundaries. Such knowledge allows for the uncertainty in system and model parameters and yields information about the expected behavior when control parameters are varied to enhance the performance of the system under consideration. In addition to providing a global-like sensitivity analysis, these methods are quite efficient in computing multiple optima. Several model problems taken from the very active area of design optimization are being investigated to test and illustrate the value and applicability of these continuation and bifurcation methods, as well as to provide motivation and focus for further development. A preliminary theory and numerical implementation have been completed as described in detail in Sections 2 and 3.

The second part centers on investigation of various numerical methods for the solution of nonlinear optimal control problems. The analysis of convergence in infinite dimensional spaces, discretizations, and numerical implementations are in progress for Newton's, penalty, augmented Lagrangian, and interior point methods. A longer term goal is the investigation of parametric problems in *nonlinear control systems* including but not limited to the nonlinear optimal control problem. Some of the initial results in this direction are described in Section 4.

The third part ot this research program is the development of combinatorial optimization techniques to solve the central problem of multi-target tracking, i.e., the data association problem of partitioning observations into tracks and false alarms. The problem formulation, algorithm design, and real-time solution involve techniques from probability and information theory, system identification, filtering, control systems, combinatorial optimization, and advanced computer architectures, including massively parallel computers. The data association problem for general multi-target tracking problems is formulated

as a class of multi-dimensional assignment problems. The algorithms under development are based on a recursive Lagrangean relaxation scheme, construct high quality suboptimal solutions in real-time, and use a variety of techniques ranging from two dimensional assignment algorithms, a conjugate subgradient method for the nonsmooth optimization, graph theoretic properties for problem decomposition, and a branch and bound technique for small solution components. These algorithms are being implemented on massively parallel computer architectures for increased performance and real-time identification. The current status and results of this research effort are described in Section 5.

The technical information concerning publications, lectures, graduate students, colleagues, and awards is contained in Section 7.

2 Predictor-Corrector Continuation Algorithms

Predictor-corrector continuation methods for tracing solution paths of an under determined nonlinear system $F(w) = 0$ where $F : \mathbb{R}^{n+1} \to \mathbb{R}^n$ have proven to be robust and effective procedures, particularly for solving problems ranging from homotopy methods for nonlinear equations, continuum mechanics, and optimization to the study of parametric dependencies in dynamical systems. (The parametric optimization problems discussed in the next section represent a primary source of problems in this work.) Although these methods are quite robust, they have been considered slow and computationally intensive, primarily because of the extensive linear algebra required in both the prediction and the correction phases. To increase the computational efficiency and robustness, a general Adams-Bashforth predictor-corrector continuation procedure, valid for both homotopy and parametric problems, has been developed by Lundberg and Poore [6]. This nonstandard ordinary differential equations technique employs a Newton-like procedure in the correction process and a variable order and an adaptive stepsize control in the prediction phase to efficiently and robustly trace out paths of solutions in these parametric problems. We [6] have showm these procedures to be highly efficient and in many respects superior to existing path following algorithms.

3 Parametric Problems in Nonlinear Programming and Control

3A Problem Statement. The parametric nonlinear programming problem is that of determining the behavior of the solution(s) as a parameter or vector of parameters $\alpha \in \mathbb{R}^r$ varies over a region of interest for the problem

$$\begin{aligned} \text{Minimize} \quad & f(x,\alpha) \\ \text{Subject To} \quad & h(x,\alpha) = 0 \qquad (3.1) \\ & g(x,\alpha) \geq 0 \end{aligned}$$

where $f : \mathbb{R}^{n+r} \to \mathbb{R}$, $\mathbb{R}^{n+r} \to \mathbb{R}^q$ and $g : \mathbb{R}^{n+r} \to \mathbb{R}^p$ are assumed to be at least twice continuously differentiable. Most applied problems contain parameters; some may be fixed but not known precisely and others may be varied to enhance the performance of the system. Good local information about the rates of change of the variables with respect to the parameters can be obtained by differentiating the Karush-Kuhn-Tucker conditions and this procedure can be rigorously justified at regular points of (3.1) by the implicit function theorem. Sensitivity is due to a "nearby" singularity in the problem, and thus we have investigated these singularities extensively [10,18,21,22]. These singularities arise from a loss of strict complementarity, a loss of the linear independence constraint qualification, or the singularity of Hessian of the Lagrangian on the tangent space to the active constraints [18].

To our knowledge, numerical continuation and bifurcation techniques have not been *systematically* developed for the fully constrained problem. These same methods, however, have been highly successful in the numerical study of dynamical systems and nonlinear equations, and thus the current objective is to develop numerical continuation and bifurcation techniques in parametric nonlinear programming in the near term and in control systems in the longer term. In addition to yielding a global-like sensitivity analysis of the parametric problem, numerical continuation and bifurcation methods also yield an effective method for computing multiple solutions.

To test and illustrate the value and applicability of our methods as well as to provide motivation and focus for further development, we are investigating several model problems taken from the very active area of design optimization. We are confident that the techniques currently under development for parametric constrained optimization problems will be very useful for investigating sensitivity, stability, and multiple optima in structural design and control problems. Our first publication on this problem is that of Lundberg and Poore [7].

3B Status of the Numerical Algorithms. To develop a numerical continuation and bifurcation approact to the parametric nonlinear programming problem, a theoretical development of the singularities in parametric nonlinear programming and a thorough understanding of a good continuation code are required. Tiahrt and Poore [10,18,21,22] have investigated singularities and persistence of the minima as well as critical point type in nonlinear parametric programming and Lundberg and Poore [6] have developed an Adams-Bashforth predictor-corrector continuation method which employs variable order and an adaptive stepsize control to efficiently and robustly trace out paths of solutions in these parametric problems. (This nonstandard ODE technique uses a Newton-like method in

the correction process.) To explain the current status of these numerical investigations, we first pose the parametric programming problem as a system of nonlinear equations [18,21]. From the Fritz John first-order necessary conditions, there exist $q + p + 1$ real numbers in ν, $\lambda = (\lambda_1, \ldots, \lambda_q)$ and $\mu = (\mu_1, \ldots, \mu_p)$, not all zero, such that

$$F(x, \nu, \lambda, \mu; \alpha) = \begin{bmatrix} \nabla_x \mathcal{L}(x, \nu, \lambda, \mu; \alpha) \\ -h(x, \alpha) \\ Mg(x, \alpha) \\ \nu^2 + \mu^T \mu + \lambda^T \lambda - \beta_0^2 \end{bmatrix} = 0, \qquad (3.2)$$

where $\mathcal{L} = \nu f(x, \alpha) - < \lambda, h(x, \alpha) > - < \mu, g(x, \alpha) >$ is the Lagrangian. In the presence of a constraint qualification, the usual normalization is to delete the last equation and set $\nu = 1$; however, the loss of the linear independence constraint qualification leads to a singularity which is generally, but not always, characterized by multipliers tending to infinity. This nonstandard normalization for the multipliers ν, λ, and μ replaces multipliers tending to infinity by $\nu \to 0$. This system system contains all solutions of the Fritz John or Karush-Kuhn-Tucker first order necessary conditions as well as minima, maxima, saddle points, and both feasible and infeasible solutions.

To explain the development, please keep in mind that singularities in the above system can only arise from a loss of strict complementarity, a loss of the linear independence constraint qualification, or the singularity of Hessian of the Lagrangian on the tangent space to the active constraints [18,21]. The easiest singularity to treat is that of the loss of strict complementarity, which corresponds to a bifurcation point whose branches can be delineated by activating and deactivating the particular constraint(s) in question. Since the signs of the inequality constraints and corresponding multipliers can be monitored to detect these bifurcations, the problem of path following and bifurcation detection in one parameter can be simplified to

$$F(x, \nu, \lambda, \hat{\mu}; \alpha) = \begin{bmatrix} \nabla_x \mathcal{L}(x, \nu, \lambda, \hat{\mu}; \alpha) \\ -h(x, \alpha) \\ -\hat{g}(x, \alpha) \\ \nu^2 + < \hat{\mu}, \hat{\mu} > + < \lambda, \lambda > - \beta_0^2 \end{bmatrix} = 0. \qquad (3.3)$$

where $\hat{g}$ and $\hat{\mu}$ are obtained from g and μ by deleting constraints and multipliers corresponding to inactive constraints. (This is essentially an "active set strategy.") Given a general predictor-corrector continuation method [6], the development of numerical continuation and bifurcation techniques for constrained optimization thus begins with the tailoring of the numerical linear algebra and singularity detection methods to this formulation of the problem.

In the continuation process, one also adds an *augmenting equation* to specify how the correction is made back to the path after a prediction, so that the coefficient matrix in the linear systems that must be solved at each point along the path are of the form

$$J = \begin{bmatrix} W & B \\ C^T & D \end{bmatrix} \quad \text{where} \quad W = \begin{bmatrix} \nabla_x^2 \mathcal{L} & -A \\ -A^T & 0 \end{bmatrix} \tag{3.4}$$

and A^T represents the derivative with respect to x of the active constraints. The last two rows and columns in J, represented by the matrices C^T, D, and B, arise from the multiplier normalization above and the augmenting equation. An effective framework for developing both the numerical linear algebra and singularity detection is a slight modification of Keller's bordering algorithm [4,5], which reduces the linear systems involving J to those involving W. Then null and range space methods and the direct factorization of W, which are extensively developed and used in optimization [1, Section 10.2], can be adapted to the continuation problem with one modification. Since in the continuation process one follows paths of all types of critical points including minima, the Hessian of the Lagrangian on the tangent space to the active constraints need no longer be positive definite. Thus the linear algebra methods need to be modified, e.g. by replacing a Cholesky factorization by a LDL^T factorization using the Bunch-Kaufman algorithm [2].

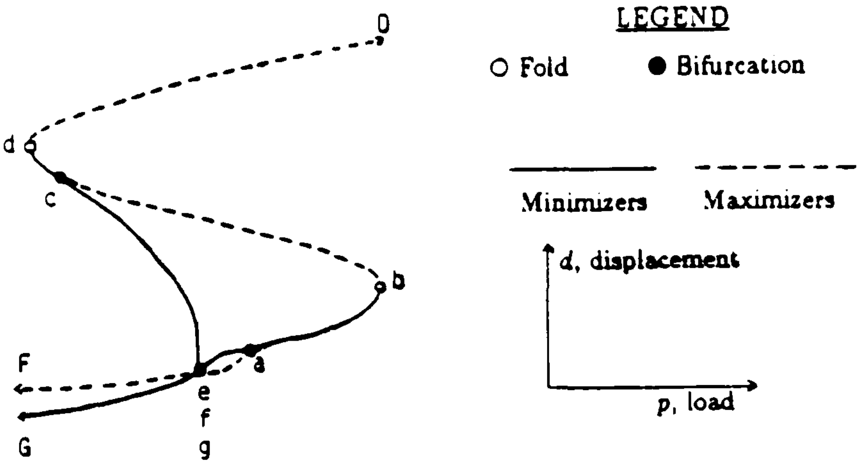
As discussed above the loss of strict complementarity can be detected by monitoring sign changes in an inactive inequality constraints and multipliers corresponding to active equality constraints. Thus methods must be developed for the remaining two singularities. Changes in the signature of the Hessian of the Lagrangian on the tangent space to the active constraints can be accomplished directly for null space methods, and using results on Schur complements [19] and closely related inertia results [3], we [8] have been able to develop efficient methods for this detection for both the range space method and the direct factorization of W. Loss of the linear independence constraint qualification occurs when $\nu \to 0$ since then there is a nonzero vector λ such that $A\lambda = 0$. We are also investigating methods based on the factorization of the constraint matrix A^T. The systematic development of these methods will be reported in a forthcoming work of Lundberg and Poore [7,8].

3C A Numerical Example from Design Optimization. As a simple illustration of the above procedures we consider the following problem from design optimization [20]:

$$\begin{aligned} &\text{Minimize} && d \\ &\text{Subject To} && \nabla_d E(d, h; p) = 0 \\ & && 0 \le h \le 1.5 \end{aligned} \tag{3.5}$$

where $E(d,h;p) = -pd + \left(\sqrt{1+h^2} - \sqrt{1+(h-d)^2}\right)^2 / \sqrt{1+h^2}$, and p is the parameter. This problem is used to model the determination of the unloaded height h of a simple two bar planar truss with semi-span 1, which minimizes the displacement d under a fixed load p. The solution paths $z(p) = (d(p), h(p), \lambda(p), \mu(p))$ of (3.5) were tracked using our continuation method. The following plot gives the displacement d as p varies.

Fig 3.1: Feasible Solutions of (3.5)



This plot represents a projection of the feasible solutions of (3.5) into the (p, d) plane. and the dot labeled with e, f and g indicates three distinct bifurcation points with $p = d = 0$ and $h = 0$, 1.41 and 1.5, respectively. Bifurcation points a, c, e and g result from a loss of strict complementarity in which an inequality constraint becomes weakly active. The path of maximizers branching from point a corresponds to $h \leq 1.5$ active and $\mu_1 < 0$. and changes type at the singular point g, becoming the path of minimizers labeled G. The other path branching from point a passes through f, across which the one eigenvalue of $\nabla_x^2 \mathcal{L}_{\bar{T}}$ changes sign. At f there is a change in type resulting in the path of maximizers labeled F.

Extreme sensitivity of the solution of (3.5) to variations in p occurs at the fold points b and d ($p = \pm.3704$) at which there is a loss of linear independence in the active constraint gradients, and $\mu_2 = 0$. This is also the case at the bifurcation point e, where in addition. strict complementarily is violated. One cannot compute near or past these points without the normalization $\nu^2 + \lambda^T\lambda + \mu^T\mu - \beta_0^2 = 0$, since near these points an unnormalized multiplier is unbounded. When the system is at a state near these points, small variations in load p can result in very large changes in the solution, or the loss of (local) existence of a solution. The latter case is illustrated near b where increasing the parameter past

$p \doteq .3704$ results in the loss of the solution and a "snap through" of the truss to a state represented by path D. Similar behavior occurs near d and e.

Not pictured above are branches of infeasible solutions of (3.5) emerging at a, c, e and g ($h > 1.5$ or $h < 0$). (In some problems such paths may provide the opportunity for further branching to other feasible paths.) A path of feasible singular points with $p = d = 0$ branches from e through f to g, and can be parameterized by μ_2. The solutions to (4.1) need only be stationary points of the potential energy $E(d, h; p)$. However, all path segments, exclusive of the segments from d to e and from c to b, do correspond to physical states of the system (where E is minimized).

Finally, note the multiple solutions in the diagram which are easily computed via these continuation procedures.

4 Nonlinear Optimal Control

The optimal control problem under consideration in this work can be described as

$$\begin{aligned}
\text{Minimize} \quad & J[x,u] := \varphi(x(t_0), x(t_1)) + \int_{t_0}^{t_1} f_0(t,x,u)dt \\
\text{Subject To} \quad & \dot{x} = f(t, x(t), u(t)) \\
& B(x(t_0), x(t_1)) = 0 \\
& h(t,x,u) = 0 \\
& g(t,x,u) \geq 0 \\
& u \in \Omega \\
& (x,u) \in W^{1,\infty}([t_0,t_1], \mathbb{R}^n) \times L^{\infty}([t_0,t_1], \mathbb{R}^m)
\end{aligned}$$

where x is an n-vector, u is an m-vector, B is a boundary operator, Ω is a closed convex set, and $W^{1,p}([t_0,t_1], \mathbb{R}^n)$ is the usual Sobolev space which can be characterized via the Sobolev imbedding theorem as consisting of those absolutely continuous vector functions with the first derivative in $L^p([t_0,t_1], \mathbb{R}^n)$. The functions φ, f_0, f, B, h, and g are assumed to be at least C^2 with respect to their arguments. In this formulation, we assume that the t_0 and t_1 are fixed; however, we stress that the more general problem in which $\varphi = \varphi(t_0, x(t_0), t_1, x(t_1))$ and the end points $(t_i, x(t_i))$ are allowed to vary can be transformed into this problem by a standard transformation that introduces two new state variables.

Our interest in this problem is two fold. First, working with W. W. Hager of the University of Florida, we are investigating the convergence of various numerical methods (Newton's, penalty, augmented Lagrangian, interior point methods) in the appropriate infinite dimensional spaces and will then work on the discretization and numerical solution

of these problems. *One paper has been submitted for publication [16], one is in preparation [17], and a third is planned.*

The second area of interest is in the parametric problem obtained by inserting parameters in the above problem. The interaction of multiple and bifurcating states in the absence of controls, periodic phenomena, chaotic behavior, and bifurcating controls arising from the dynamical systems and holonomic constraints is open to investigation. Given a certain phenomena arising from a dynamical system, the problem may be to control this phenomena, to determine multiple solutions, or to investigate the dependence of a solution on the system parameters over a wide range, i.e. global sensitivity. (The latter is also important in adaptive control.) The development and use of theoretical and numerical bifurcation and continuation methods in dynamical systems and nonlinear equations has been spectacularly successful in analyzing and understanding the phenomena represented by these systems, but we know of no systematic treatment or works on the *constrained* nonlinear parametric control problem parallelling that found in dynamical systems. Thus a long term goal of this research program will be the investigation of parametric problems in *nonlinear control systems* including but not limited to the nonlinear optimal control problem.

5 Combinatorial Optimization and Multi-Target Tracking

5A Problem Statement.

The third part of this research program is the development of combinatorial optimization techniques to solve the central problem of multi-target tracking, i.e., the data association problem of partitioning observations into tracks and false alarms. Although combinatorial optimization is the natural framework for the formulation of these problems, the corresponding techniques have long been considered computationally too intensive for real-time applications and for good reason. The resulting optimization problems, which are formulated here as multi-dimensional assignment problems, are NP-hard. To further appreciate the difficulties, one only has to examine the trade-offs between two current methods in multiple target tracking: track while scan and batch. For the former, one essentially extends tracks a scan at a time using for example a two dimensional assignment or a greedy algorithm. This methodology is real-time, but results in a large number of partial and incorrect assignments, and thus incorrect track identification. The fundamental difficulty with this approach is there is simply not enough information in one scan at a time processing to properly partition the observations into tracks and false alarms. To obtain the required information, one needs to consider several scans all at once, i.e. the batch approach, but it is this batch approach that is considered computationally too inten-

sive for practical real-time applications. Given the ever-present need to identify *all* tracks in *real-time*, the challenge to combinatorial optimization is to design fast algorithms for advanced computer architectures that will solve the underlying data association problem, and thus the track identification, in real-time.

What we have accomplished is the real-time or near real-time of many data association problems by exploiting sparsity and problem decomposition, by developing a Lagrangian relaxation algorithm for the construction of 'high quality' suboptimal solutions, and by using advanced computer architectures such as massively parallel architectures. In the subsections to follow, the problem formulation, algorithms, current results, and parallel computing issues are discussed.

5B Physical Model. A number of objects or targets, say 100 - 10,000, are assumed to obey some underlying physics from which one can formulate physical laws for the equations of motion. Most models for real-time application are based on linear systems theory wherein *each* target is assumed to obey an equation of the form

$$\frac{dx}{dt} = F(t)x + \Gamma(t)u + G(t)w$$

$$z = H(t)x + v$$

where $F(t)$, $\Gamma(t)$, $G(t)$, $H(t)$ are assumed to be known, x is the state variable, u is the input or control function, w is the input or process noise, v is the observation error, and z is the output or observed quantity. What differentiates one target from another might be initial state values or parameters in $F(t)$, $\Gamma(t)$, $G(t)$, $H(t)$; these time dependent matrices can also vary from target to target in a more general way.

At a set of *scan* times $\{t_k\}_{k=1}^{K}$ pictures of the objects are taken, and the *observations* are recorded as $\{z_{i_k}^k\}_{i_k=0}^{N_k}$ for scan time t_k. Here, K represents the number of scans.) N_k is the number of observations on scan k, and the zero index, $i_k = 0$, corresponds to a dummy or missed observation.

Given this description, the *data association problem* is that of partitioning the observations into tracks and false alarms [11,12] in such a way that the paths or tracks of the objects can be identified. Smoothing, filtering, and prediction techniques are then used to obtain further information about past, present, and future states of the objects.

5C Mathematical Formulation of the Data Association Problem. In what follows, the term *track of observations* is used to denote a sequence of observations $\{z_{i_k}^k\}_{k=1}^{K}$, one from each scan, that might be generated by the target or object. Since the potential

number of tracks of observations is too large to consider computationally, a *gating* procedure is employed to remove unlikely tracks of observations and thus introduce sparsity into the problem [11].

Given a track of observations $\{z_{i_k}^k\}_{k=1}^K$, the *filtering problem* is to estimate the state up to the current time t_k. The first use of filtering and system identification is in the development of a score function since measurement error in the track of observations is scored against the filtered track. Specifically, Kalman filtering, recursive system identification, and adaptive filtering techniques are particularly relevant to this problem.

Using K scans of information, the next task is to formulate a *K-dimensional assignment problem* whose solution gives an optimal partitioning of observations into tracks and false alarms. Given a track of observations $(z_{i_1}^1, \ldots, z_{i_K}^K)$, define the 0-1 variable

$$z_{i_1,\ldots,i_K} = \begin{cases} 1 & \text{if } (z_{i_1}^1, \ldots, z_{i_K}^K) \text{ is assigned to a track,} \\ 0 & \text{otherwise.} \end{cases}$$

The score of the assignment of the observations $(z_{i_1}^1, \ldots, z_{i_K}^K)$ to a track is defined to be

$$c_{i_1,i_2,\ldots,i_K} = \begin{cases} -ln(p_{i_1,i_2,\ldots,i_K}) & \text{if } (z_{i_1}^1, \ldots, z_{i_K}^K) \text{ passes gating,} \\ \infty & \text{if } (z_{i_1}^1, \ldots, z_{i_K}^K) \text{ fails gating,} \end{cases}$$

where $p_{i_1,i_2,\ldots,i_K}$ is a composite probability density function involving probabilities for measurement error, false alarms, missed detections, track life, initiation, and finite sensor resolution [13].

The problem of assigning 0 or 1 to all the variables $z_{i_1,\ldots,i_K}$ in such a way that each actual observation is assigned to exactly one track total cost is minimized is called the *assignment problem* which can be formulated as

$$\begin{aligned}
\text{Minimize} \quad & v(z) = \sum_{i_1=0}^{N_1} \cdots \sum_{i_K=0}^{N_K} c_{i_1,\ldots,i_K} z_{i_1,\ldots,i_K} \\
\text{Subj. To} \quad & \sum_{i_2=0}^{N_2} \cdots \sum_{i_K=0}^{N_K} z_{i_1,\ldots,i_K} = 1, \quad i_1 = 1, \ldots, N_1, \\
& \sum_{i_1=0}^{N_1} \cdots \sum_{i_{k-1}=0}^{N_{k-1}} \sum_{i_{k+1}=0}^{N_{k+1}} \cdots \sum_{i_K=0}^{N_K} z_{i_1,\ldots,i_K} = 1, \quad i_k = 1, \ldots, N_k \\
& \sum_{i_1=0}^{N_1} \cdots \sum_{i_{K-1}=0}^{N_{K-1}} z_{i_1,\ldots,i_K} = 1, \quad i_K = 1, \ldots, N_K \\
& z_{i_1,\ldots,i_K} \in \{0,1\}, \quad \forall i_1, \ldots, i_K.
\end{aligned}$$

An important feature of this formulation is that the number of targets is not required *a priori* and is determined as part of the solution process.

5D Algorithms for the Construction of Real-Time Solutions. The basic scheme currently used employs preprocessing in the form of gating and problem decomposition. Then the sparse decomposed problems are solved by a recursive Lagrangean relaxation scheme. A K-dimensional assignment problem is relaxed to a $(K-1)$-dimensional one by incorporating one set of constraints into the objective function using a Lagrangean relaxation of this set. Given a solution of the $(K-1)$-dimensional problem, a feasible solution of the K-dimensional problem is then reconstructed. The $(K-1)$-dimensional problem is solved in a similar manner and the process is repeated until one reaches the two-dimensional problem which is solved exactly. The duality gap in this process is generally quite small and one obtains in general an ϵ-optimal solution. The full technical description can be found in the forth coming paper of Poore and Rijavec [11].

5E A Case Study. The following three tables illustrate the solution quality, current timings, and expected timings for the identification of 100 tracks consisting of straight lines in two dimensional space-time. The two tables below present a numerical comparison for the straight lines whose intercepts and slopes are uniformly distributed [0,1000] and [-0.2,0.2], respectively. The observation errors are assumed to be Gaussian random variables with a zero mean and standard deviation σ. In the following tables the *maximum error* of 3σ is defined as the mean distance between the tracks at the initial time and ranges between 1% and 50%. The scan times are spaced 40 seconds apart. These problems are scale invariant for $m_{max}\Delta t = 0.2 \times 40$ where Δt denotes the time between scans. Thus if the observations are taken every 5 seconds, the slopes can range between -1.6 and 1.6. All computations were performed on Silicon Graphics Personal IRIS, and twenty (20) test problems were randomly generated and solved to obtain the averaged results given in the two tables below. Thus an identification reading of 99.95% implies that all but one track out of 2000 was correctly identified.

% max. error	3 scans	4 scans	5 scans	6 scans	7 scans	8 scans
1.0	100.00	100.00	100.00	99.90	100.00	100.00
5.0	99.30	99.60	99.80	99.85	100.00	99.95
10.0	97.10	99.15	99.80	99.65	99.85	99.70
20.0	95.75	98.20	98.85	98.85	99.05	99.25
30.0	94.50	96.35	97.90	97.70	98.80	98.75
40.0	92.10	94.20	96.30	97.50	97.30	99.25
50.0	93.50	94.20	95.60	95.10	97.25	-

Solution Quality: % of Tracks Correctly Identified

The maximum error of 1% corresponds to a high signal to noise ratio, whereas 50% corresponds to a very low signal to noise ratio. The identification is of exceptional quality over all signal to noise ratios and gradually improves as one moves across the table. The reason for incomplete identification is that one encounters regions of high contention where many tracks cross. This difficulty is resolved *locally* on subsequent scans, but other regions appear. In the next table the solution times are presented.

% max. error	3 scans	4 scans	5 scans	6 scans	7 scans	8 scans
1.0	0.05	0.05	0.06	0.06	0.07	0.07
5.0	0.19	0.31	0.42	0.59	1.01	1.61
10.0	0.66	1.31	1.94	3.24	5.01	6.55
20.0	1.25	3.32	7.15	16.60	27.86	53.45
30.0	1.96	6.01	18.10	52.58	118.15	276.58
40.0	2.14	8.50	28.93	90.69	281.15	506.11
50.0	2.37	11.03	42.98	156.76	379.62	–

Current Solution Times in Seconds

We have mentioned real-time in this work; let us now be specific. Suppose a radar sweep takes 5 to 10 seconds. The objective then is to process as many scans as possible between sweeps to improve identification and solve the problem in the alloted time. The above table gives some idea of the capability at the present for 100 targets. To appreciate these timings further, one must consider the several factors that can significantly improve the speed. The research code based on the current algorithms is designed for adaptability, not speed. (A special purpose three and four dimensional assignment code ran between six and ten times faster than our recursive K-dimensional code used to generate these numbers.) Perhaps the most significant speedup can be achieved by what has not been used. Given a specific K dimensional assignment problem, one most often has a suboptimal or optimal solution of a closely related K or $K-1$ dimensional problem and such a solution should be used as a hot start. For example, in going from k scans of observations to $k+1$ scans, almost all tracks have been identified, but we are not making use of these hot starts. The following table gives the expected solution times for this problem.

% max. error	3 scans	4 scans	5 scans	6 scans	7 scans	8 scans
1.0	0.01	0.01	0.01	0.01	0.01	0.01
5.0	0.02	0.03	0.04	0.04	0.05	0.05
10.0	0.05	0.07	0.09	0.09	0.10	0.15
20.0	0.07	0.10	0.15	0.20	0.20	0.30
30.0	0.10	0.15	0.30	0.50	0.80	1.20
40.0	0.10	0.20	0.40	0.60	1.40	3.00
50.0	0.10	0.20	0.50	1.20	3.00	4.00

Expected Solution Times in Seconds

5F Parallel Implementations. The multi-target tracking algorithms are in the process of being parallelized on a massively parallel SIMD architecture. The goal has been to investigate and understand the issues and algorithms in a parallel computing environment. The machine chosen for algorithm development and testing was the Connection Machine CM2 at the National Center for Atmospheric Research (NCAR) in Boulder, Colorado. The Connection Machine consists of a large number (up to 64K) of 1-bit serial processors, each with 64K bits of local memory. For the purpose of this project, only a small number of the Connection Machine processors were used, configured into a linear array as requested by IBM of Owego, New York. The association of regions of space to processors and then the load balancing for efficiency will not be discussed, since problem formulation is still under development.

Two of the main issues in implementing algorithms on a massively parallel issues are balancing the computational loads across the processors and minimizing the communication paths. To balance computational loads we have developed a dynamic load balancing strategy that is purely local in nature, i.e. does not assume the existence of the front end computer. Various improvement of this algorithm, depending on a particular machine characteristics, are being investigated. Next, a comprehensive strategy for fitting the tracking problem to a massively parallel SIMD architecture has been developed. This strategy starts with dividing the target space into slices that are then allocated to individual processors. This strategy is consistent with the objective of minimizing communication. As an added benefit, the computing loads arising from such allocation are more likely to be balanced, making the load balancing algorithms cheaper to execute.

The issue of parallel indexing (i.e. indexing parallel arrays by parallel indices) has turned out to be fundamentally important for efficient implementation of tracking algorithms on a massively parallel SIMD architecture. Most algorithms require at least some parallel indexing, while some are very heavily dependent upon it. The architecture that

does not provide an efficient parallel indexing will not be well suited for these tracking algorithms.

Specifically some of the modules that have been implemented todate include:

a *A tracking model to generate the observations.* This model is implemented in parallel and incorporates targets moving with constant speeds in one dimensional space, target initiation and termination at any time, probability of detection of less then one, false alarms (clutter) and finite sensor resolution.

b *Problem formulation module that formulates a multi-dimensional assignment problem on the basis of observations and known model parameters.*

c *A decomposition algorithm to identify disjoint components of the assignment problem.*

d *A branch and bound algorithm for small multi-dimensional assignment problems.*

e *The auction algorithm of Bertsekas for the two dimensional assignment problems.*

f *A dynamic load balancing algorithm.*

g *Various sorting and utility modules.*

Our current thinking leads to the *conclusion* that the preferred architecture for implementing multi-target tracking algorithms is a shared memory MIMD architecture with large number of processors, e.g., BBN Butterfly. However, the algorithms are also well suited to implementation on a massively parallel linear SIMD machine. Such machines are much less complex and thus likely to be both smaller in size and more economical to build.

6 References

1. Fletcher, R., *Practical Methods of Optimization*, Second Edition, John Wiley and Sons, New York, 1987.
2. Golub, G. H. and C. G. Van Loan, *Matrix Computations*, Second Edition, The John Hopkins University Press, Baltimore, 1989.
3. Jongen, H. Th., T. Mobert, J. Ruckmann, and K. Tammer, On Inertia and Schur Complement in Optimization, *Linear Algebra and Its Applications*, 95 (1987), 97-109.
4. Keller, H. B., The bordering algorithm and path following near singular points of higher nullity, *SIAM J. Sci. and Statist. Comp.*, 4 (1983), pp 573-582.
5. Keller, H. B., Global homotopies and Newton methods, in C. De Boor and G. H. Golub, eds.,*Recent Advances in Numerical Analysis*, Academic Press, New York, 1978. pp. 73-94.
6. B. N. Lundberg and A. B. Poore, Variable order Adams-Bashforth predictors with error-stepsize control for continuation methods, to appear in SIAM J Sci. and Stat. Comp.
7. B. N. Lundberg and A. B. Poore, Bifurcations and Sensitivity in Parametric Nonlinear Programming, in V. B. Venkayya, J Sobieski, and L. Berke, eds., *Proceedings of the Third Air Force/NASA Symposium on Recent Advances in Multidisciplinary Analysis and Optimization*, (1990) pp 50-55.
8. B. N. Lundberg and A. B. Poore, Continuation Techniques for Parametric Nonlinear Programming, in preparation.
9. B. N. Lundberg, A. B. Poore, and B. Yang, Smooth Penalty Functions and Continuation Methods for Constrained Optimization, in E. L. Allgower and K. Georg, editors, *Lectures in Applied Mathematics Series*, Volume 26, American Mathematical Society, Providence, pp 389-412, 1990.
10. A. B. Poore, Bifurcations in Parametric Nonlinear Programming, to appear in *Annals of Operations Research, Optimization with Data Perturbations*, 1990.
11. A. B. Poore and N. Rijavec, A Lagrangean Relaxation Algorithm for Multidimensional Assignment Problems Arising from Multi-target Tracking, submitted 1990.
12. A. B. Poore and N. Rijavec, A New Class of Methods for Solving Data Association Problems Arising from Multi-target Tracking, submitted 1990.
13. A. B. Poore and N. Rijavec, Multidimensional Assignment Problems Arising from Multiple Target Tracking, in preparation.
14. A. B. Poore and N. Rijavec, Combinatorial Optimization in Multi-target Tracking, in preparation (with N. Rijavec).

15. A. B. Poore and N. Rijavec, The Data Association Problem Arising from Multi-target Tracking, in preparation.

16. A. B. Poore, B. Yang, and W. W. Hager, Penalty, Multiplier, and Newton Methods for a Class of Nonlinear Optimal Control Problems, submitted 1990.

17. A. B. Poore, B. Yang, and W. W. Hager, Convergence Analysis of a Class of Numerical Methods for Nonlinear Optimal Control, in preparation.

18. A. B. Poore and C. A. Tiahrt, Bifurcation problems in nonlinear parametric programming, *Mathematical Programming*, 39 (1987) 189-205.

19. D. V. Quellette, Schur complements and statistics, *Linear Algebra and Its Applications*, 36 (1981) 187-295.

20. M. Save and W. Prager, eds., *Structural Optimization*, Vol. 2, Plenum Press, New York, 1990.

21. C. A. Tiahrt and A. B. Poore, A bifurcation analysis of the nonlinear parametric programming problem, *Mathematical Programming*, 47 (1990) 117-141.

22. C. A. Tiahrt and A. B. Poore, Bifurcation and Persistence of Minima in Nonlinear Parametric Programming, in K Bowers and J Lund, editors, *Progress in Systems and Control Theory: Computation and Control*, Birkhauser, Boston, 1989, pp 343-354.

7 Technical Information

7A Honors and Awards in 1990

1. The Burlington-Northern Foundation Faculty Achievement Award for Research and Graduate Education (1990).
2. Appointed to the editorial board of *Computational Optimization and Control, An International Journal.*

7B Participating Professionals and Graduate Students

Marc Munger and Joe Persicketti, IBM-SID, Boulder, CO.
Tom Barker and Richard Blahut, IBM-FSD, Owego, NY.
Tom Depkovich, Martin Marietta Astronautics Group, Denver, CO.
Bruce N. Lundberg, Dept. of Mathematics, Grand Canyon University, Phoenix, AZ.
William W. Hager, University of Florida, Gainesville, FL.

Graduate Students at Colorado State University

Mohammed Hasan (Phd 1991)	Layne Hellrickson (MS 1990)
Bing Yang (PhD 1991)	Jackie Hoover (MS 1990)
Nenad Rijavec (PhD 1992)	Michael Williams (MS 1990)

7C Publications in 1990

1. A Bifurcation Analysis of the Nonlinear Parametric Programming Problem, Mathematical Programming, 47 (1990) 117-141 (with Chris A. Tiahrt).
2. Smooth Penalty Functions and Continuation Methods for Constrained Optimization, in E. L. Allgower and K. Georg, editors, Lectures in Applied Mathematics Series, Volume 26, American Mathematical Society, Providence, pp 389-412, 1990. (With B.N. Lundberg and B. Yang)
3. A Tubular Chemical Reactor Model, in E. L. Allgower and K. Georg, editors, Lectures in Applied Mathematics Series, Volume 26, American Mathematical Society, Providence, pp 749-752, 1990.
4. Bifurcations in Parametric Nonlinear Programming, Annals of Operations Research, Optimization with Data Perturbations, 1990.
5. Bifurcations and Sensitivity in Parametric Nonlinear Programming, in V. B. Venkayya, J Sobieski, and L. Berke, eds., Proceedings of the Third Air Force/NASA Symposium on Recent Advances in Multidisciplinary Analysis and Optimization, (1990) pp 50-55, with Bruce N. Lundberg.
6. Penalty, Multiplier, and Newton Methods for a Class of Nonlinear Optimal Control Problems, submitted 1990 (with B. Yang and W. W. Hager).
7. A Lagrangean Relaxation Algorithm for Multi-dimensional Assignment Problems Arising from Multi-target Tracking, submitted 1990 (with N. Rijavec).

8. A New Class of Methods for Solving Data Association Problems Arising from Multi-target Tracking, submitted 1990 (with N. Rijavec).

9. Multidimensional Assignment Problems Arising from Multiple Target Tracking, in preparation (with N. Rijavec).

10. Combinatorial Optimization in Multiple Target Tracking, in preparation (with N. Rijavec).

11. The Data Association Problem Arising from Multi-target Tracking, in preparation (with N. Rijavec).

12. Convergence Analysis of a Class of Numerical Methods for Nonlinear Optimal Control, in preparation, with B. Yang and W. W. Hager.

13. Continuation Techniques for Parametric Nonlinear Programming, in preparation, with Bruce N. Lundberg.

7D Lectures in 1990

1. Multi-Target Tracking and Assignment Problems: First Quarter Presentation, IBM-SID, Boulder, CO, April, 1990.

2. Combinatorial Optimization Approaches to Multi-Target Tracking, IBM Federal Systems Division, Owego, NY, April, 1990.

3. Multi-Target Tracking and Assignment Problems: Second Quarter Presentation, IBM-SID, Boulder. CO, June, 1990.

4. The Data Association Problem in Multi-Target Tracking, BDM International, Boulder, CO, June, 1990.

5. The Data Association Problem in Multi-Target Tracking, LOGICON, Colorado Springs, CO, June, 1990.

6. Penalty-Multiplier Methods for Nonlinear Optimal Control Problems, Bozeman Conference on Numerics and Control, Bozeman, Montana, August, 1990.

7. Multi-Target Tracking and Assignment Problems: Third Quarter Presentation, IBM-SID, Boulder, CO, September, 1990.

8. Sensitivity and Bifurcations in Parametric Nonlinear Programming, NASA/Air Force Symposium, San Francisco, CA, September, 1990.

9. Multi-Target Tracking and Assignment Problems, Hughes Aircraft, El Segundo, CA, November 5,1990.

10. Multiplier-Continuation Methods for Nonlinear Optimal Control, SIAM Conference on Linear Algebra, Signals, Systems, and Control, San Francisco, CA, November 7, 1990, Invited 30 Min. Presentation, with B. Yang and W. W. Hager.

11. Multi-Target Tracking and Assignment Problems: 1990 Final Presentation IBM-SID, Boulder, CO, December 13,1990.